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p	q	m	n
3	2	1	1
3	2	2	3
2	1	1	1
2	31	5	1
2	127	7	1

ERRATUM. In 156 for e^2 read e^3.

AVERAGE AND PROBABILITY.

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196. Proposed by R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a square. Find the chance that the distance between two points within the square and without the circle shall not exceed a side of the square.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $2a$=side of square; (x, y), (u, v) the coordinates of the points; $\sqrt{(a^2-x^2)}=y_1$, $\sqrt{(a^2-u^2)}=v_1$. If both points are in the same corner, the distance between them is always less than $2a$. If both points are situated one each in opposite corners, the distance between them is always greater than $2a$. If both points are placed one each in adjacent corners, we have $\sqrt{[(x-u)^2+(y+v)^2]}=4a^2$, for the greatest distance between the points and $\sqrt{\{(x-u)^2+[\sqrt{(a^2-x^2)}+\sqrt{(a^2-u^2)}]^2\}}=\sqrt{[(x-u)^2+(y+v)^2]}$, for the least distance between the points.

$$\therefore v_2=\sqrt{[4a^2-(x-u)^2]}-y,\ v_3=\sqrt{(a^2-x^2)}+\sqrt{(a^2-u^2)}-y.$$

$$\therefore p=\frac{\int_0^a\int_0^a\left[\int_{y_1}^a\int_{v_1}^a dydv+\int_{y_1}^a\int_{v_1}^{v_2}dy\,dv\right]dxdu+\int_0^a\int_0^x\int_{y_1}^a\int_{v_3}^{v_2}dxdudydv}{3\int_0^a\int_0^a\int_{y_1}^a\int_{v_1}^a dx\,dy\,du\,dv}$$

$$=\frac{a^4(4-\pi)^2+0+.16\int_0^a\int_0^x\int_{y_1}^a\int_{v_3}^{v_2}dx\,du\,dy\,dv}{3a^4(4-\pi)^2}=\tfrac{1}{3}+\frac{1.6}{3a^4(4-\pi)^2}.M.$$

$$M=\int_0^a\int_0^x\int_{y_1}^a\{\sqrt{[4a^2-(x-u)^2]}-\sqrt{[a^2-x^2]}-\sqrt{[a^2-u^2]}\}dx\,du\,dy$$

$$=\int_0^a\int_0^x[a-\sqrt{(a^2-x^2)}]\{\sqrt{[4a^2-(x-u)^2]}-\sqrt{[a^2-x^2]}-\sqrt{[a^2-u^2]}\}dx\,du$$

$$=\int_0^a [a-\sqrt{(a^2-x^2)}]\left(\frac{x}{2}\sqrt{[4a^2-x^2]}-\tfrac{3}{2}x\sqrt{[a^2-x^2]}+2a^2\sin^{-1}\frac{x}{2a}\right.$$

$$\left.-\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right)dx=a^4\int_0^{\frac{1}{2}\pi}(1-\cos\theta)\,[\tfrac{1}{2}\sin\theta\sqrt{(4-\sin^2\theta)}-\tfrac{3}{2}\sin\theta\cos\theta$$

$$+2\sin^{-1}(\tfrac{1}{2}\sin\theta)-\tfrac{1}{2}\theta]\cos\theta\,d\theta=a^4(\tfrac{3}{2}\sqrt{3}-\tfrac{29}{12}+\frac{\pi^2}{32}+\frac{\pi}{12})$$

$$-\,a^4\int_0^{\frac{1}{2}\pi}[\tfrac{1}{2}\sin\theta\sqrt{\,)4-\sin^2\theta)}+2\sin^{-1}(\tfrac{1}{2}\sin\theta)]\cos^2\theta\,d\theta$$

$$=a^4(\tfrac{3}{2}\sqrt{3}-\tfrac{29}{12}+\frac{\pi^2}{32}+\frac{\pi}{12})-a^4\int_0^{\frac{1}{2}\pi}(2\sin\theta-\tfrac{1}{8}\sin^2\theta+\frac{15}{2^7}\sin^3\theta-\frac{1}{2^{10}}\sin^4\theta$$

$$+\frac{763}{2^{15}}\sin^5\theta-\frac{7}{2^{18}}\sin^6\theta+\frac{10219}{2^{22}}\sin^7\theta-\ldots)\cos^2\theta\,d\theta.$$

$$\therefore M=a^4(\tfrac{3}{2}\sqrt{3}-\tfrac{29}{12}+\frac{\pi^2}{32}+\frac{\pi}{12}-0.6595495)=0.0412517a^4.$$

$\dfrac{16M}{3a^4(4-\pi)^2}=\dfrac{0.6600272}{2.2108850}=\tfrac{3}{10}$, nearly, $=.299$. $\therefore p=\tfrac{1}{3}+\tfrac{3}{10}=\tfrac{19}{30}$, nearly.

PROBLEMS FOR SOLUTION.

ALGEBRA.

208. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Find the conditions that the roots of $x^2+px+q=0$ may not lie between -1 and $+1$.

209. Proposed by PROFESSOR E. B. ESCOTT, Ann Arbor, Mich.

Solve, $bx^2+cy^2+az^2=ba^2+cb^2+ac^2$,
$cx^2+ay^2+bz^2=ab^2+bc^2+ca^2$,
$xyz=abc$.

210. Proposed by B. F. FINKEL, Ph. D., Drury College, Springfield, Mo.

Simplify, $\log[\sqrt[3]{(137)}\;\sqrt[7]{(56)}\div\sqrt[10]{(187)}\;\sqrt[4]{(75)}]$.